Instructor: **Grøndahl**

Name: Key

MATH 173
Unit Exam 4

Show All Work
Justify All Conclusions
No Graphing Calculators Allowed

1) Use the table and graph below to
   a. Calculate $f(g(5))$

$f(0)=2$

   b. Solve $g(g(x))=3$

$x=4.75$

$g(g(4.75))=g(.75)=3$

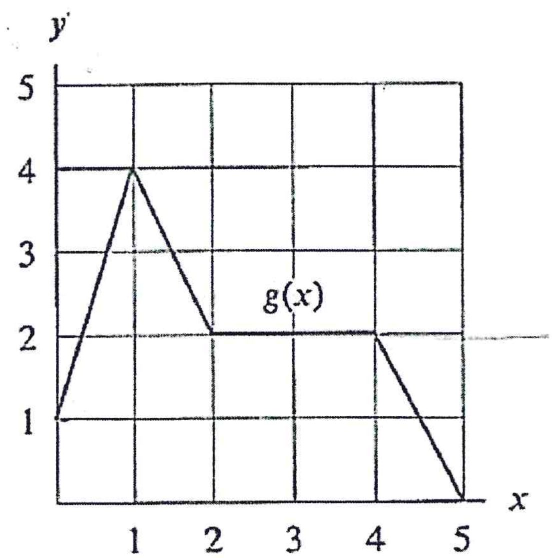
$x=0.25$

$g(g(0.25))=g(1.5)=3$

$x=4.25$

$g(g(4.25))=g(1.5)=3$

x	0	1	2	3	4	5
$f(x)$	2	5	3	4	1	0



2) Find a formula for the inverse of the function $k(x)=\dfrac{2+\sqrt{x}}{\sqrt{x}-1}$

$$X=\frac{2+\sqrt{k^{-1}(x)}}{\sqrt{k^{-1}(x)}-1}$$

$$x\left(\sqrt{k^{-1}(x)}-1\right)=2+\sqrt{k^{-1}(x)}$$

$$x\sqrt{k^{-1}(x)}-\cancel{x}=2+\sqrt{k^{-1}(x)}$$

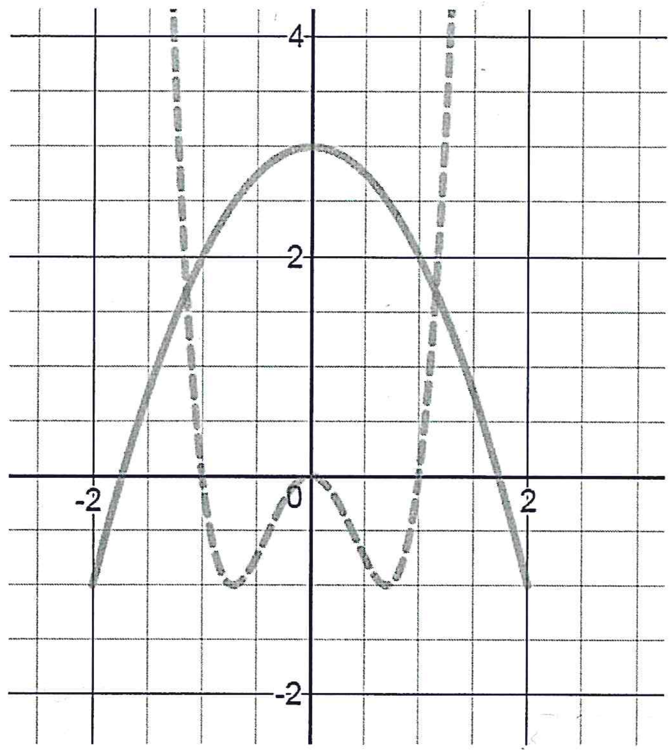
$$x\sqrt{k^{-1}(x)}-\sqrt{k^{-1}(x)}=x+2$$

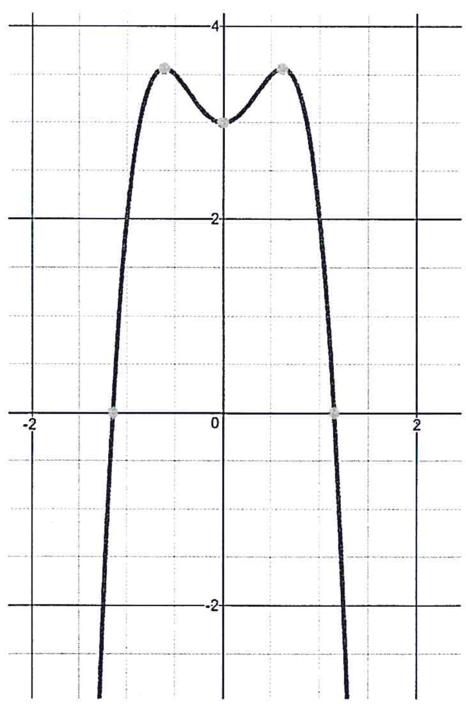
$$\sqrt{k^{-1}(x)}\,(x-1)=x+2$$

$$\sqrt{k^{-1}(x)}=\frac{x+2}{x-1}$$

$$k^{-1}(x)=\left(\frac{x+2}{x-1}\right)^2$$

3) Below are the graphs of $f(x)$ shown as a solid line and $g(x)$ shown as a dashed line. Sketch the graph of $h(x) = f(x) - g(x)$





4) Describe the behavior of $f(x)=x^{-3}$ and $g(x)=x^{1/3}$

a. As $x \to 0$ from the right

$f(x) \to \infty$

$g(x) \to 0$

b. As $x \to \infty$

$f(x) \to 0$

$g(x) \to \infty$

5) Match each function with its graph

a. $f(x)=0.3x^4-2x^2+17$

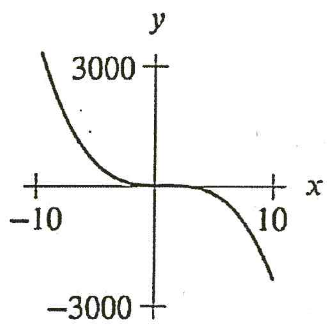
b. $g(x)=-3x^3+7x^2-2x+5$

c. $h(x)=4x^5+2x^3-7x-5$

d. $k(x)=-8x^6-4x^3+12x^2$

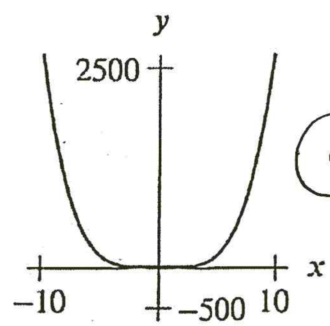
(I)

b



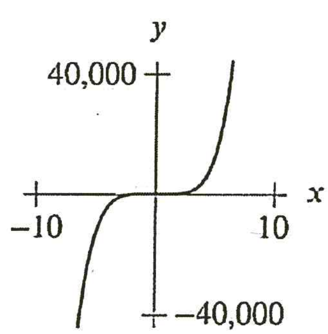
(II)

a



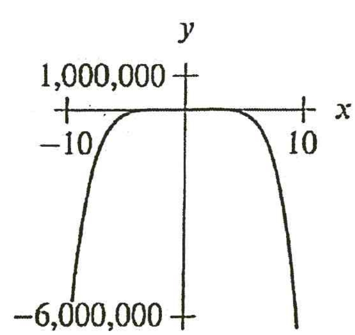
(III)

c

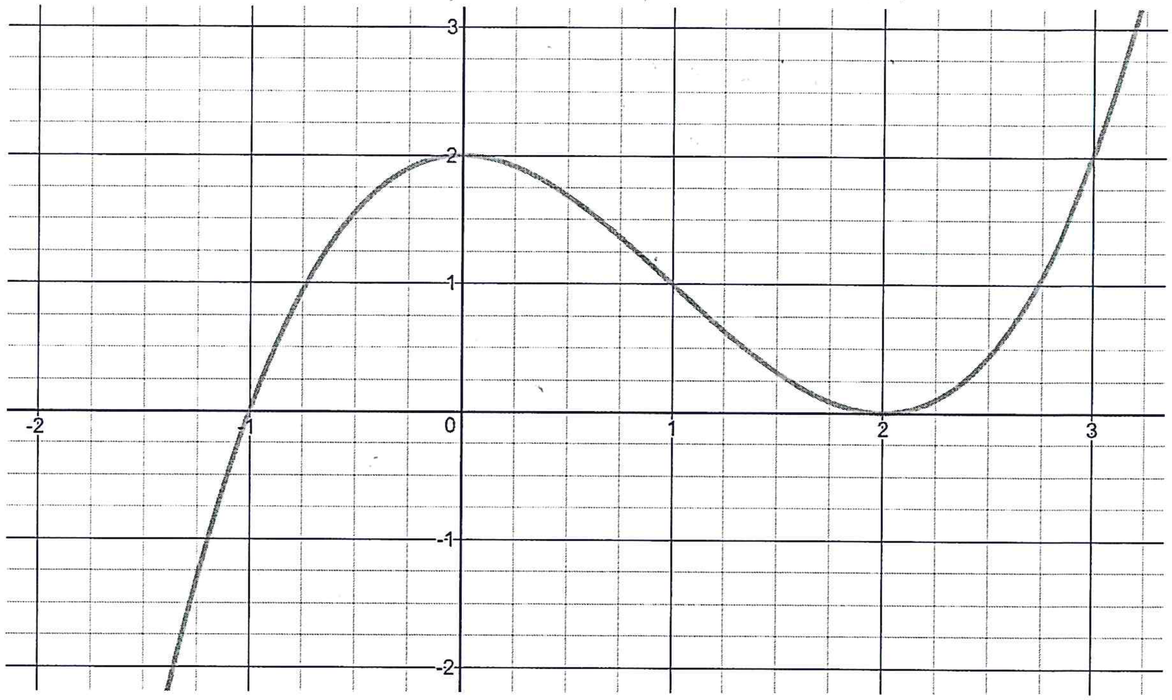


(IV)

d



6) Find a possible formula for $g(x)$ that is a third degree polynomial with graph,



$$g(x) = c(x+1)(x-2)^2$$

$$g(0) = c(1)(4) = 2$$

$$c = 1/2$$

$$g(x) = \frac{1}{2}(x+1)(x-2)^2$$

7) Factor $P(x) = 3x^3 + 7x^2 + 7x + 3$ over the complex numbers.

$$
\begin{array}{r|l}
 & 3x^2 + 4x + 3 \\
\hline
x+1 & 3x^3 + 7x^2 + 7x + 3 \\
 & -(3x^3 + 3x^2) \\
\hline
 & 4x^2 + 7x \\
 & -(4x^2 + 4x) \\
\hline
 & 3x + 3 \\
 & -(3x + 3) \\
\hline
 & 0
\end{array}
$$

$$P(x) = (x+1)(3x^2+4x+3)$$

$$x = \frac{-4 \pm \sqrt{16-4(9)}}{2(3)}$$

$$= \frac{-4 \pm \sqrt{-20}}{6}$$

$$= \frac{-2 \pm 2i\sqrt{5}}{3}$$

$$P(x) = 3(x+1)\left(x + \frac{2}{3} + \frac{2}{3}i\sqrt{5}\right)\left(x + \frac{2}{3} - \frac{2}{3}i\sqrt{5}\right)$$

8) If $f(x) = x^5$ find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ and determine what the difference quotient approaches as $h \to 0$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$$= \frac{(x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5) - x^5}{h}$$

$$= 5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4, \quad h \neq 0$$

$$\to 5x^4 \quad \text{As} \quad h \to 0$$

9) A chemist is studying the properties of a bronze alloy of copper and tin. She begins with 6 kg of an alloy that is one-half tin. Keeping the amount of copper constant, she adds small amounts of tin to the alloy. Letting x be the total amount of tin added, define,

$$C(x) = \text{Concentration of Tin} = \frac{\text{Amount of Tin}}{\text{Amount of Alloy}}$$

a) Find a formula for C(x)

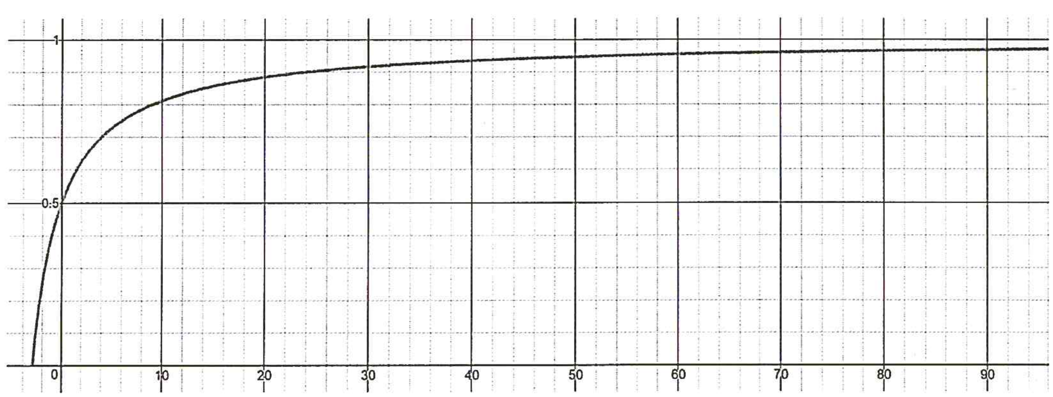
$$C(x) = \frac{3+x}{6+x}, \quad x \geq -3$$

b) Evaluate C(2) and C(−6). Explain the physical significance of these quantities.

$C(2) = 5/8$ IS THE PROPORTION OF TIN IN THE ALLOY AFTER AN ADDITIONAL 2 kg OF TIN HAVE BEEN ADDED

$C(-6)$ DNE BECAUSE THERE WAS ONLY 3 kg OF TIN TO START WITH SO YOU CANNOT REMOVE 6 kg

c) Graph y=C(x)



10) Without a calculator, match the functions to the graphs.

a. $y = \dfrac{-1}{(x-5)^2} - 1$

b. $y = \dfrac{(x-2)}{(x+1)(x-3)}$

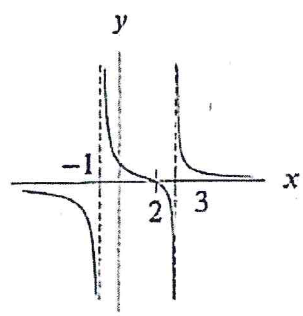
c. $y = \dfrac{2x+4}{x-1}$

d. $y = \dfrac{1}{x+1} + \dfrac{1}{x-3}$

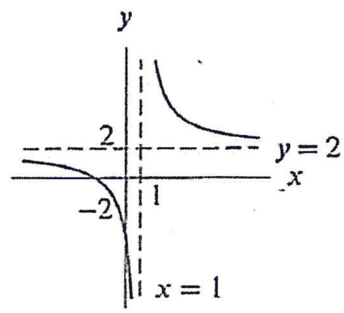
e. $y = \dfrac{1-x^2}{x-2}$

f. $y = \dfrac{1-4x}{2x+2}$

(b) (i)

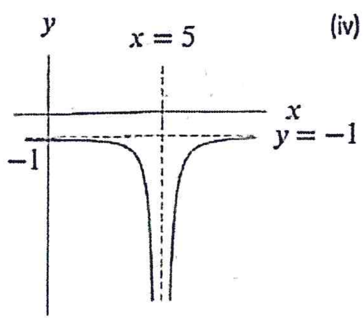


(ii)

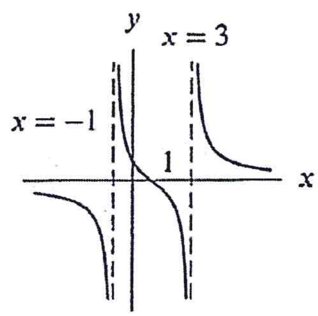


(c)

(a) (iii)

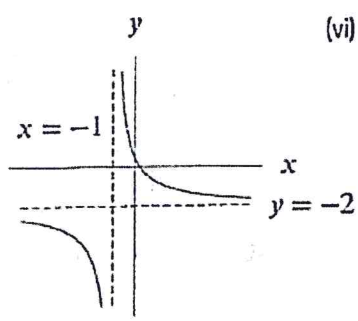


(iv)

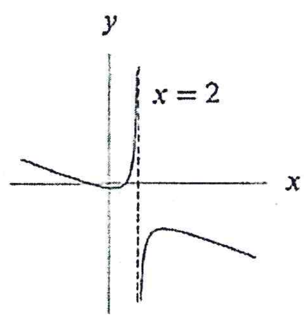


(d)

(f) (v)



(vi)



(e)

11) Find a possible formula for f, where $f(1)=8$ and $f(2)=64$, if f is

a. Linear

$$m = \frac{64-8}{2-1} = 56$$

$$f(x) - 8 = 56(x-1)$$

$$f(x) = 56x - 48$$

b. Exponential

$$f(x) = ab^x$$

$$ab^1 = 8$$

$$a = \frac{8}{b}$$

$$\frac{8}{b} b^2 = 64$$

$$b = 8$$

so $a = \frac{8}{8} = 1$

$$f(x) = 1(8)^x$$

c. Power Function

$$f(x) = ax^n$$

$$8 = a1^n$$

$$a = 8$$

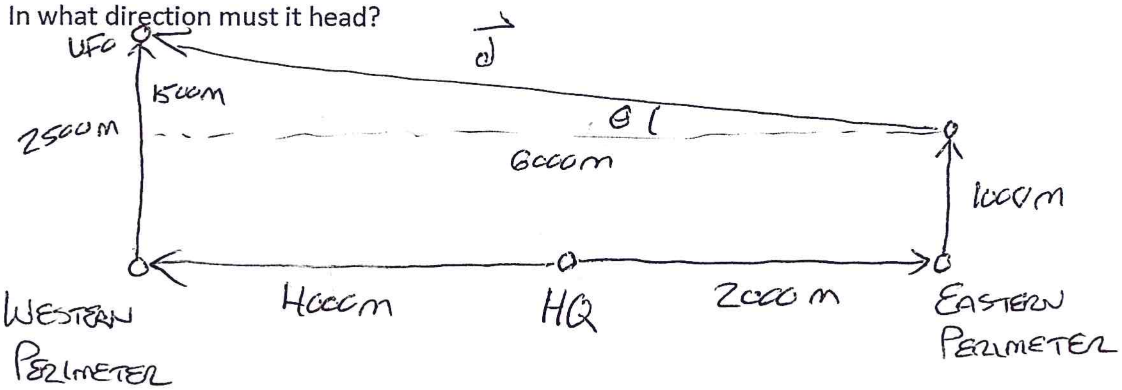
$$64 = 8(2)^n$$

$$8 = 2^n$$

$$3 = n$$

$$f(x) = 8x^3$$

12) A helicopter is hovering 1000 meters directly over the eastern perimeter of a secret Air Force installation. The eastern perimeter is 2000 meters straight east from the installation headquarters. The helicopter receives a transmission from headquarters that a UFO has been sighted at 2500 meters directly over the installation's western perimeter, which is 4000 meters straight west from the headquarters. How far must the helicopter travel to intercept the UFO? In what direction must it head?



$$\theta = \tan^{-1}\left(\frac{1500}{6000}\right) \approx 14^\circ$$

$$|\vec{d}| = \sqrt{6000^2 + 1500^2} \approx 6185m$$

13) Which is traveling faster, a car with a velocity of $\vec{v}_1 = \langle 5, -6 \rangle$ m/s, or a car with a velocity vector of $\vec{v}_2 = \langle -6, 4 \rangle$ m/s?

$$|\vec{v}_1| = \sqrt{25+36} = \sqrt{61}\ m/s$$

$$|\vec{v}_2| = \sqrt{36+16} = \sqrt{52}\ m/s$$

$$|\vec{v}_1| = \sqrt{61}\ m/s > \sqrt{52}\ m/s = |\vec{v}_2|$$

14) Three different electric charges q_1, q_2 and q_3 exert forces on a test charge, Q . The forces are, respectively, $\vec{F}_1 = \langle 3,-7 \rangle$, $\vec{F}_2 = \langle -1,4 \rangle$, and $\vec{F}_3 = \langle 8,7 \rangle$

a. Calculate $\vec{F}_{\text{Net}}$

$$\vec{F}_{NET} = \langle 3-1+8, -7+4+7 \rangle$$

$$= \langle 10, 4 \rangle$$

b. If a forth charge is added, q_4, with an associated force of $\vec{F}_4$, what must this force be so that Q experiences a net force of zero?

$$\vec{F}_4 = -\vec{F}_{NET} = \langle -10, -4 \rangle$$

15) Calculate the magnitude of the acceleration due to gravity experienced by an astronaut in the ISS. The ISS orbits 400727 meters above the Earth's surface. The Earth's radius is 6.371×10^{6} meters. The Earth's mass is 5.972×10^{24} kg.

Newton's Second Law: $\vec{F} = m\vec{a}$

Gravitational Force: $\vec{F} = -\left(G\dfrac{m_1 m_2}{|\vec{r}|^2}\right)\dfrac{\vec{r}}{|\vec{r}|}$

Universal Gravitational Constant: $G = 6.67\times10^{-11}\,\mathrm{m^3/kg\cdot s^2}$

Note: All units given agree, so no conversion in needed. Acceleration will be in $\mathrm{m/s^2}$

$$\vec{a} = \frac{\vec{F}}{m} = -\left(\frac{Gm_1}{|\vec{r}|^2}\right)\frac{\vec{r}}{|\vec{r}|}$$

$$|\vec{a}| = \frac{Gm_1}{|r|^2} = \frac{(6.67\times10^{11})(5.972\times10^{24})}{(6.371\times10^{6}+400727)^2}$$

$$\approx 8.7\ \mathrm{m/s^2}$$