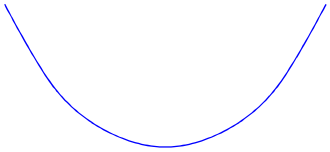
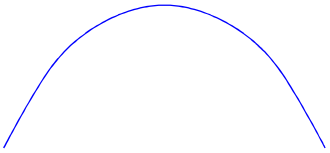
Section 2.5 – Concavity

Definitions.

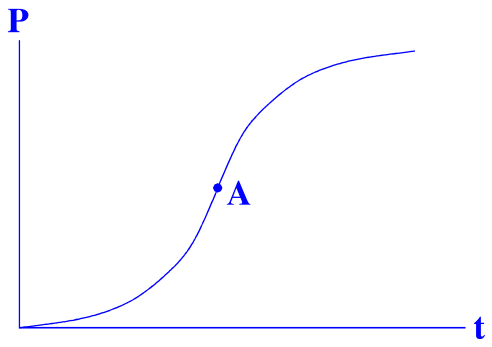
1. A function $f(x)$ is called *increasing* if its graph __rises__ from left to right. It is called *decreasing* if its graph __falls__ from left to right.
2. A function $f(x)$ is called *concave up* if its average rate of change increases from left to right.
3. A function $f(x)$ is called *concave down* if its average rate of change decreases from left to right.

Describe the shape of the graph of a function $f(x)$ that is concave up:	Describe the shape of the graph of a function $f(x)$ that is concave down:
Concave up means shaped like part or all of a right-side up bowl: 	Concave down means shaped like part or all of an upside-down bowl: 

Example. Read the following description of a function. Then, decide whether the function is increasing or decreasing. What does the scenario tell you about the concavity of the graph modeling it?

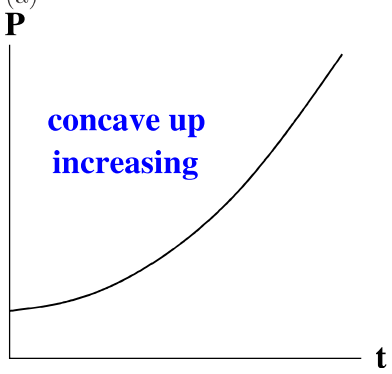
> "When a new product is introduced, the number of people who use the product increases slowly at first, and then the rate of increase is faster (as more and more people learn about the product). Eventually, the rate of increase slows down again (when most people who are interested in the product are already using it)."

Let t represent time, and let P represent the number of people using the product at time t. The function is increasing because, as time goes on, more and more people use the product. The function is concave up until the point labeled "A" and is concave down thereafter.

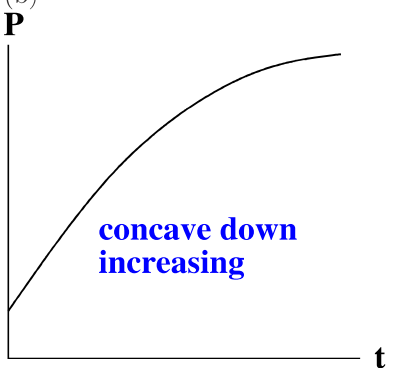


Example. Consider the following graphs of population, P, as a function of time, t.

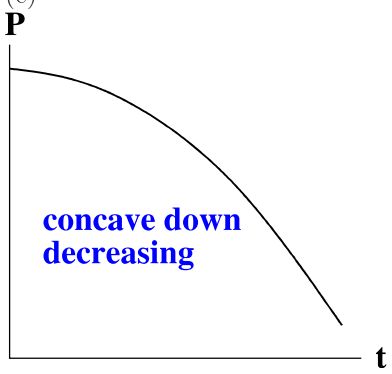
(a)



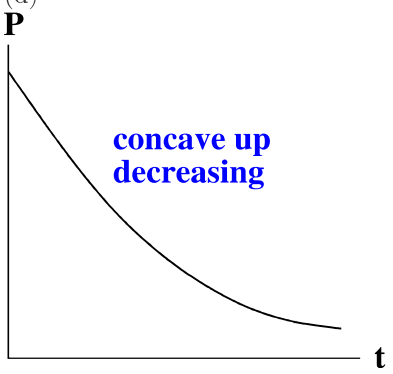
(b)



(c)



(d)



Descriptions

(a) P is increasing, and the rate of change of P is increasing.

(b) P is increasing, and the rate of change of P is decreasing.

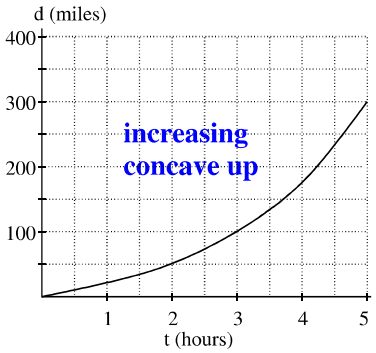
(c) P is decreasing, and the rate of change of P is decreasing.

(d) P is decreasing, and the rate of change of P is increasing.

Examples and Exercises

1. Consider the functions shown below. Fill in the accompanying tables and then decide whether each function is increasing or decreasing, and whether it is concave up or concave down.

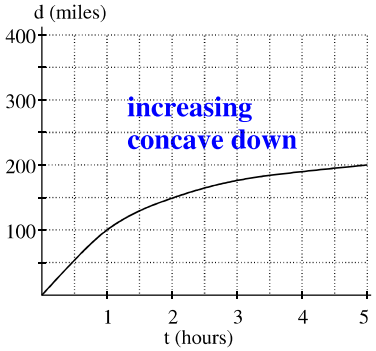
(a) **Description.** This graph gives distance driven as a function of time for a California driver.



t	0	2	3	5
d	0	50	100	300
$\frac{\Delta d}{\Delta t}$	$\frac{50}{2}$ $= 25$	$\frac{50}{1}$ $= 50$	$\frac{200}{2}$ $= 100$ mi/hr	

(rates are increasing)

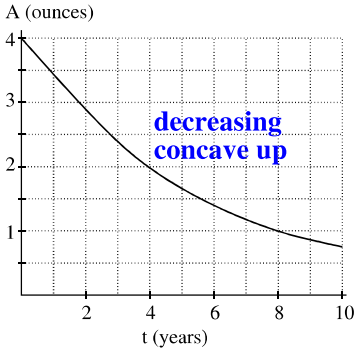
(b) **Description.** This graph gives distance driven as a function of time for a South Dakota driver.



t	0	2	3	5
d	0	150	175	200
$\frac{\Delta d}{\Delta t}$	$\frac{150}{2}$ $= 75$	$\frac{25}{1}$ $= 25$	$\frac{25}{2}$ $= 12.5$ mi/hr	

(rates are decreasing)

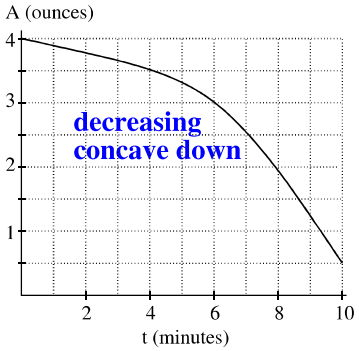
(c) **Description.** This graph gives the amount of a decaying twinkie as a function of time.



t	0	4	6	10
A	4	2	1.4	0.75
$\frac{\Delta A}{\Delta t}$	$\frac{-2}{4}$ $= -0.5$	$\frac{-0.6}{2}$ $= -0.3$	$\frac{-0.65}{4}$ $= -0.1625$ oz/yr	

(rates are increasing)

(d) **Description.** This graph gives the amount of ice remaining in a melting ice cube as a function of time.



t	0	4	6	10
A	4	3.5	3	0.5
$\frac{\Delta A}{\Delta t}$	$\frac{-0.5}{4}$ $= -0.125$	$\frac{-0.5}{2}$ $= -0.25$	$\frac{-2.5}{4}$ $= -0.625$	

(rates are decreasing)

2. Decide whether each of the following functions are concave up, concave down, or neither.

x	0	1	2	3	4
$f(x)$	1	3	6	10	20

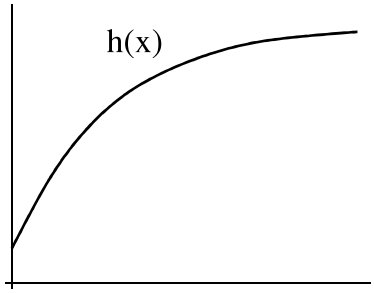
Concave up because average rates of change increase from left to right

x	0	1	2	3	4
$g(x)$	10	9	7	4	0

Concave down because average rates of change decrease from left to right.

$p(x) = 3x + 1$

Neither, because the rate of change for a linear function is constant everywhere.



Concave down