Section 2.5 – Preview of Inverse Functions

Preliminary Example. Recall the phone example from earlier, where a calling plan charged us a \$30 monthly service fee and then \$0.10 per minute for long distance calls.

t	0	30	33	36	60
C	30	33	33.30	33.60	36

$$\begin{aligned} C &= f(t) \longleftarrow \texttt{Cost, } C\texttt{, as a function of time, } t \\ t &= f^{-1}(C) \longleftarrow \texttt{Time, } t\texttt{, as a function of cost, } C \end{aligned}$$

For each of the following, fill in the blank and then give an interpretation of the entire statement.

(a) $f(36) =$ 33.60

If you talk for 36 minutes of long distance, your phone bill is \$33.60.

(b) $f^{-1}(36) =$ 60

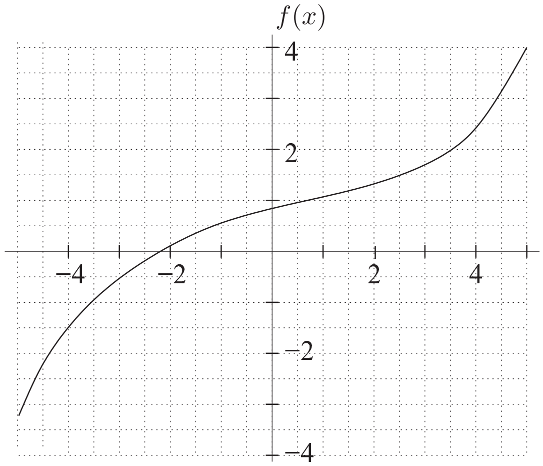
If your phone bill is \$36, then you talked for 60 minutes of long distance.

(c) $f^{-1}($ 33.30 $) = 33$

Your phone bill is \$33.30 if you talk 33 minutes of long distance.

Examples and Exercises

1. Use the two functions shown below to fill in the blanks to the right.



x	-6	-4	-2	0	2	4	6
$g(x)$	2	0	3	7	6	1	5

(a) $f(2) =$ 1.3 (b) $f^{-1}(2) =$ 3.5

(c) $g(0) =$ 7 (d) $g^{-1}(0) =$ −4

(e) $f(3) + 1 =$ 2.7 (f) $f^{-1}(3) + 1 =$ 5.4

(g) $f(3 + 1) =$ 2.4 (h) $f^{-1}(3 + 1) =$ 5

(i) If $g^{-1}(x) = 0$, then $x =$ 7.

2. Let $A = f(n)$ be the amount of periwinkle blue paint, in gallons, needed to paint n square feet of a house. Explain in practical terms what each of the following quantities represents. Use a complete sentence in each case.

   (a) $f(20)$

   This represents the amount of paint, in gallons, needed to paint 20 square feet of a house.

   (b) $f^{-1}(20)$

   This represents the number of square feet of a house that can be painted with 20 gallons of paint.

3. If a cricket chirps R times per minute, then the outside temperature is given by $T = f(R) = \frac{1}{4}R + 40$ degrees Fahrenheit.

(a) Find a formula for the inverse function $R = f^{-1}(T)$.

$$\begin{aligned} T &= \frac{1}{4}R + 40 & \longleftarrow T \texttt{ as a function of } R \\ 4T &= R + 160 & \\ R &= 4T - 160 & \longleftarrow R \texttt{ as a function of } T \end{aligned}$$

We conclude from the work above that $f^{-1}(T) = 4T - 160$.

(b) Calculate and interpret $f(50)$ and $f^{-1}(50)$.

First, we have

$$f(50) = \frac{1}{4}(50) + 40 = 12.5 + 40 \quad = \quad 52.5.$$

This indicates that 52.5°F is the temperature at which a cricket chirps 50 times per minute. Next, we have

$$f^{-1}(50) = 4(50) - 160 = 200 - 160 \quad = \quad 40.$$

This indicates that when the temperature is 50°F, a cricket will chirp 40 times per minute. To summarize, our numerical answers are $f(50) = \$52.5$ and $f^{-1}(50) = 40$ chirps per minute.