Section 2.2 – Domain and Range

Definition. If $Q = f(t)$, then

1. The ___domain___ of f is the set of all input values, t, that yield a meaningful output value.
2. The ___range___ of f is the corresponding set of all output values.

Example 1. Let $A = f(r)$ be the area, in cm^2, of a circle of radius r cm. Find the domain and the range of f.

$$\begin{aligned} \text{Domain of } f &= \{r : r > 0\} \\ \text{Range of } f &= \{A : A > 0\} \end{aligned}$$

Comment: We know from geometry that $f(r) = \pi r^2$. Note that the domain is not all real numbers because nonpositive values of r don't make sense in the context of this function.

Example 2. Find the domain and range of the function $f(x) = \sqrt{x+2}$.

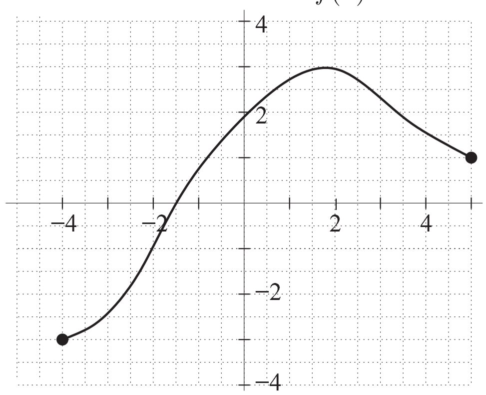
Since we can only take square roots of nonnegative real numbers, we know that

$$x + 2 \geq 0 \quad \Longrightarrow \quad x \geq -2,$$

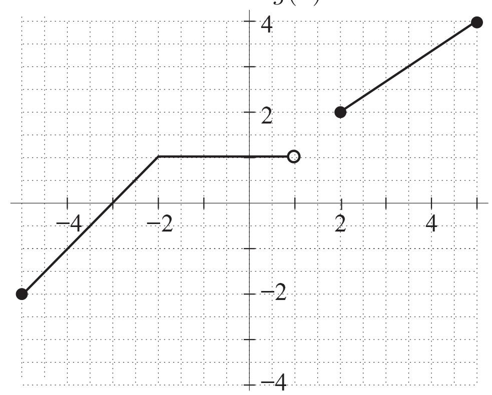
so the domain of f is $\{x : x \geq -2\}$. Using a graph as an aid, we see that the range of f is $\{y : y \geq 0\}$.

1. For each of the following functions below, give the domain and the range.

$f(x)$



$g(x)$



Domain: $\{x : -4 \leq x \leq 5\}$
Range: $\{y : -3 \leq x \leq 3\}$

Domain: $\{x : -5 \leq x < 1\} \cup \{x : 2 \leq x \leq 5\}$
Range: $\{y : -2 \leq x \leq 1\} \cup \{y : 2 \leq y \leq 4\}$

2. Oakland Coliseum is capable of seating 63,026 fans. For each game, the amount of money that the Raider's organization makes is a function of the number of people, n, in attendance. If each ticket costs \$30.00, find the domain and range of this function. Sketch its graph.

Let r represent the revenue that the Raider's organization makes, so that $r = f(n)$. Since n represents a number of people, it must be a nonnegative whole number. Therefore, since 63,026 is the maximum number of people who can attend a game, we can describe the domain of f as follows:

$$\text{Domain} = \{n : 0 \leq n \leq 63{,}026 \text{ and } n \text{ is an integer}\}$$

The range of the function consists of all possible amounts of revenue that could be earned. To explore this question, note that $r = 0$ if nobody comes to the game, $r = 30$ if one person comes to the game, $r = 60$ if two people come to the game, etc. Therefore, r must be a multiple of 30 and cannot exceed $30 \cdot 63{,}026 = 1{,}890{,}780$, so we see that

$$\text{Range} = \{r : 0 \leq r \leq 1{,}890{,}780 \text{ and } r \text{ is a multiple of } 30\}.$$

3. Find the domain and range of each of the following functions.

(a) $f(x) = \sqrt{3x+7}$

Since

$$3x + 7 \geq 0 \implies 3x \geq -7 \implies x \geq -\frac{7}{3},$$

the domain of f is $\{x : x \geq -7/3\}$. Using a graph, we see that the range is given by $\{y : y \geq 0\}$.

(b) $g(x) = \dfrac{1}{(x-1)^2}$

Since

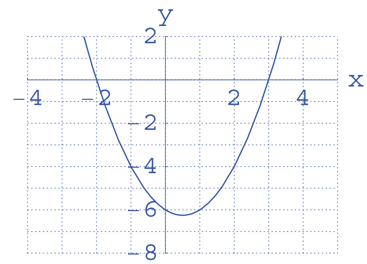
$$(x-1)^2 = 0 \implies x - 1 = 0 \implies x = 1,$$

we see that $x = 1$ is the only value of x that produces a zero denominator. Therefore, the domain of f is all real numbers except 1, or $\{x : x \neq 1\}$. A graph reveals that the range is $\{y : y > 0\}$.

(c) $h(x) = x^2 - x - 6$

Since we can legally substitute any value in for x, the domain of h is all real numbers. A graph reveals that the range of h is $\{y : y \geq -6.25\}$.

$y = x^2 - x - 6$



(d) $k(x) = \sqrt{x^2 - x - 6}$

In order for $k(x)$ to be defined, we need $x^2 - x - 6 \geq 0$. Referring to the graph above, we see that this happens if $x \geq 3$ or if $x \leq -2$. Therefore, the domain of k is $\{x : x \leq -2 \text{ or } x \geq 3\}$. As in part (a), the range of k is $\{y : y \geq 0\}$.