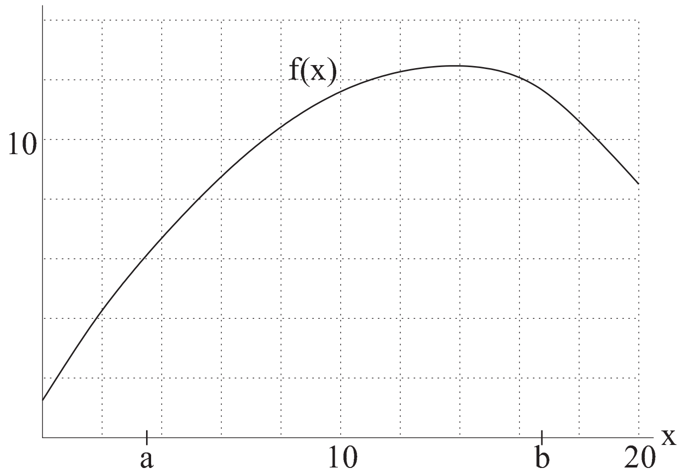
Section 2.1 – Input and Output

Preliminary Example. Complete each of the following.



1. $f(10) =$ 11.5.
2. If $f(x) = 10$, then $x =$ 7.2 or 18.7.
3. $f(a) =$ 6.
4. $f(10) - f(6) =$ 11.5 – 8.7 = 2.8.

Examples and Exercises

1. The following table shows the amount of garbage produced in the U.S. as reported by the EPA.

t (years: $1960 \equiv 60$)	60	65	70	75	80	85	90
G (millions of tons of garbage)	90	105	120	130	150	165	180

Consider the amount of garbage G as a function of time $G = f(t)$. Include units with your answers.

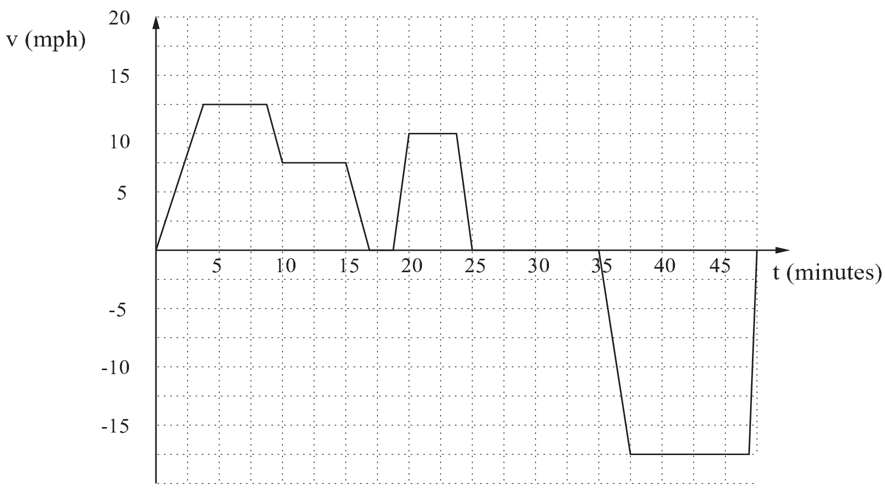
(a) $f(60) =$ 90 million tons of garbage

(b) $f(75) =$ 130 million tons of garbarge

(c) Solve $f(t) = 165$.

$t = 85$ years, which represents the year 1985.

2. Given is the graph of the function $v(t)$. It represents the velocity of a man riding his bike to the library and going back home after a little while. A negative velocity indicates that he is riding toward his house, away from the library.



Evaluate and interpret:

(a) $v(5) =$ 12.5 m.p.h.

After 5 minutes, the man is riding at a speed of 12.5 miles per hour toward the library.

(b) $v(40) = -17.5$ m.p.h.

After 40 minutes, the man is riding at a speed of 17.5 miles per hour toward home.

(c) $v(12) - v(7) = 7.5 - 12.5 = -5$ m.p.h.

The man is riding 5 miles per hour faster toward the library after 7 minutes than he is after 12 minutes.

Solve for t and interpret:

(d) $v(t) = 5$

Solutions: $t = 1.5,\ 16,\ 19,\ 24$ minutes

These are the times when the man is riding at a speed of 5 miles per hour toward the library.

(e) $v(t) = -10$

Solutions: $t = 37,\ 47$ minutes

These are the times when the man is riding at a speed of 10 miles per hour toward home.

(f) $v(t) = v(10)$

Solutions: $t = 2.5,\ 19.5,\ 23.5,$ and all real numbers between 10 and 15 minutes

These are the times when the man is riding at the same velocity as he is when $t = 10$ minutes.

3. Consider the functions given below.

(a) Let $f(x) = x^2 - 2x - 8$.

i. Find $f(0)$.

$f(0) = 0^2 - 2 \cdot 0 - 8 = -8.$

ii. Solve $f(x) = 0$.

We have

$$\begin{aligned} x^2 - 2x - 8 = 0 \quad &\implies \quad (x-4)(x+2) = 0 \\ &\implies \quad x = 4 \text{ or } x = -2, \end{aligned}$$

so our answers are 4 and -2.

(b) Let $f(x) = \dfrac{1}{x+2} - 1$

i. Find $f(0)$.

$f(0) = \dfrac{1}{0+2} - 1 = -\dfrac{1}{2}.$

ii. Solve $f(x) = 0$.

We have

$$\begin{aligned} \frac{1}{x+2} - 1 = 0 \quad &\implies \quad \frac{1}{x+2} = 1 \\ &\implies \quad 1 = x + 2 \\ &\implies \quad -1 = x, \end{aligned}$$

so our answer is -1.

4. Let $f(x) = \dfrac{x}{x+1}$. Calculate and simplify $f\left(\dfrac{1}{t+1}\right)$, writing your final answer as a single fraction.

We have

$$\begin{aligned} f\left(\frac{1}{t+1}\right) = \frac{\frac{1}{t+1}}{\frac{1}{t+1} + 1} \quad &= \quad \frac{\frac{1}{t+1}}{\frac{1}{t+1} + \frac{t+1}{t+1}} \\ &= \quad \frac{\frac{1}{t+1}}{\frac{1+t+1}{t+1}} \\ &= \quad \frac{\frac{1}{t+1}}{\frac{1+t+1}{t+1}} \cdot \frac{t+1}{t+1} \\ &= \quad \frac{1}{1+t+1}, \end{aligned}$$

so our final answer is $\dfrac{1}{t+2}$.