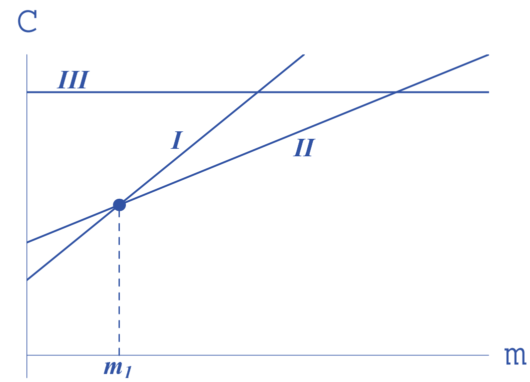
Sections 1.4 & 1.5 – Geometric Properties and Modeling with Linear Functions

Example 1. You need to rent a car for one day and to compare the charges of 3 different companies. Company I charges \$20 per day with an additional charge of \$0.20 per mile. Company II charges \$30 per day with an additional charge of \$0.10 per mile. Company III charges \$70 per day with no additional mileage charge.

(a) For each company, find a formula for the cost, C, of driving a car m miles in one day. Then, graph the cost functions for each company for $0 \le m \le 500$. (Before you graph, try to choose a range of C values would be appropriate.)

`Total Cost = Daily Charge + (Charge Per Mile)(Miles Driven)`

$$\begin{aligned} C &= 20 + 0.20m \quad \longleftarrow \texttt{Company I} \\ C &= 30 + 0.10m \quad \longleftarrow \texttt{Company II} \\ C &= 70 \quad \longleftarrow \texttt{Company III} \end{aligned}$$



(b) How many miles would you have to drive in order for Company II to be cheaper than Company I?

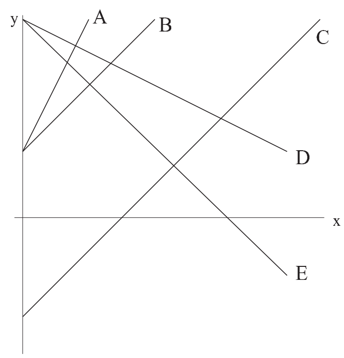
From the graph in part (a), we see that Company II is cheaper if we drive more than m_1 miles. We therefore begin by finding the point of intersection of lines I and II:

$$20 + 0.20m = 30 + 0.10m \quad \Longrightarrow \quad -10 = -0.10m \quad \Longrightarrow \quad 100 = m,$$

Thus, $m_1 = 100$ miles, and we conclude that Company II is cheaper if we drive more than 100 miles.

Example 2. Given below are the equations for five different lines. Match each formula with its graph to the right.

- $f(x) = 20 + 2x$ $\longleftarrow$ B
- $g(x) = 20 + 4x$ $\longleftarrow$ A
- $h(x) = 2x - 30$ $\longleftarrow$ C
- $u(x) = 60 - x$ $\longleftarrow$ D
- $v(x) = 60 - 2x$ $\longleftarrow$ E



Facts about the Line $y = mx + b$

1. The y-intercept, b (also called the vertical intercept), tells us where the line crosses the y-axis.
2. If $m > 0$, the line rises left to right. If $m < 0$, the line falls left to right.
3. The larger the value of $|m|$ is, the steeper the graph.

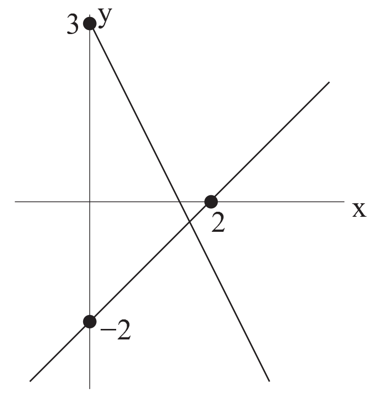
Parallel and Perpendicular Lines

Fact: Two lines ($y = m_1x + b_1$ and $y = m_2x + b_2$) are ...

1. ...parallel if $m_1 = m_2$ (slopes are the same).
2. ...perpendicular if $m_1 = -\dfrac{1}{m_2}$ (slopes are negative reciprocals of one another).

Examples and Exercises

1. Consider the lines given in the figure to the right. Given that the slope of one of the lines is -2, find the **exact coordinates** of the point of intersection of the two lines. ("Exact" means to leave your answers in fractional form.)



First, note that the slope of the line having a positive slope goes through the points $(0, -2)$ and $(2, 0)$, so its slope is

$$m = \frac{0-(-2)}{2-0} = 1,$$

and we can see that the y-intercept is -2; therefore, this line has equation $y = x - 2$. We are given that the slope of the other line is -2, and we can see that its y-intercept is 3; therefore, its equation is given by $y = -2x + 3$.

To solve the problem, we must find the point of intersection of the lines $y = x - 2$ and $y = -2x + 3$. Thus, we have

$$x - 2 = -2x + 3 \quad \Longrightarrow \quad 3x = 5 \quad \Longrightarrow \quad x = \frac{5}{3},$$

and so $y = x - 2 = \frac{5}{3} - 2 = -\frac{1}{3}$. Our final answer is therefore $(\frac{5}{3}, \ -\frac{1}{3})$.

2. Parts (a) and (b) below each describe a linear function. Find a formula for the linear function described in each case.

(a) The line parallel to $2x - 3y = 2$ that goes through the point $(1, 1)$.

First, note that

$$2x - 3y = 2 \quad \Longrightarrow \quad -3y = 2 - 2x$$
$$\Longrightarrow \quad y = \frac{2}{3}x - \frac{2}{3},$$

so the given line has slope $2/3$. Therefore, using point-slope form for the equation of a line, we have

$$y - 1 = \frac{2}{3}(x - 1) \quad \Longrightarrow \quad y = \frac{2}{3}x - \frac{2}{3} + 1$$
$$\Longrightarrow \quad y = \frac{2}{3}x + \frac{1}{3}.$$

Thus, our final answer is $y = \frac{2}{3}x + \frac{1}{3}$.

(b) The line perpendicular to $2x - 3y = 2$ that goes through the point $(1, 1)$.

As in part (a), the given line $2x - 3y = 2$ has slope $2/3$. Therefore, the slope of a perpendicular line is $-3/2$. Once again, using point-slope form, we have

$$y - 1 = -\frac{3}{2}(x - 1) \quad \Longrightarrow \quad y = -\frac{3}{2}x + \frac{3}{2} + 1$$
$$\Longrightarrow \quad y = -\frac{3}{2}x + \frac{5}{2},$$

so our final answer is $y = -\frac{3}{2}x + \frac{5}{2}$.