Section 1.1 – Functions and Function Notation

Definition. A *function* is a rule that takes certain values as inputs and assigns to each input value exactly one output value.

Example. Let $y = \dfrac{2}{1+x}$.

x	0	1	2	3
y	2	1	$\frac{2}{3}$	$\frac{1}{2}$

y as a function of x: $f(0) = 2,\ f(1) = 1,\ f(2) = 2/3,\ f(1/2) = 3$

x as a function of y: $g(2) = 0,\ g(1) = 1,\ g(2/3) = 2,\ g(1/2) = 3$

Example.

t = time (in years) after the year 2000

w = number of San Francisco '49er victories

t	0	1	2	3	4	5	6	7	8	9
w	6	12	10	7	2	4	7	5	7	8

Observations:

1. w is a function of t:

$f(0) = 6$ means that in 2000, the '49ers won 6 games

$f(1) = 12$ means that in 2001, the '49ers won 12 games

$f(2) = 10$ means that in 2002, the '49ers won 10 games

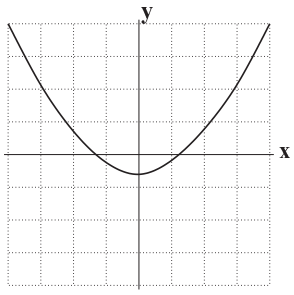
$\vdots$

2. t is not a function of w, since the input "7 wins" would have three outputs, 3, 6, and 8.

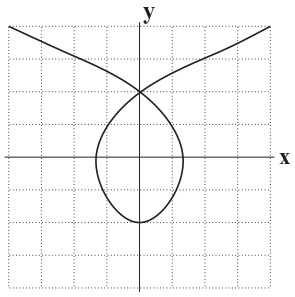
3. In general, for a function, the input variable is called the *independent* variable and the output variable is called the *dependent* variable.

Example. Which of the graphs below represent y as a function of x?

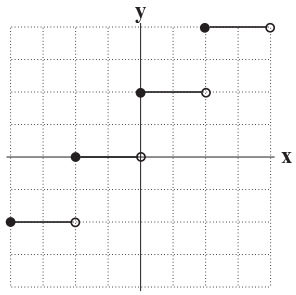
Graph 1



Graph 2

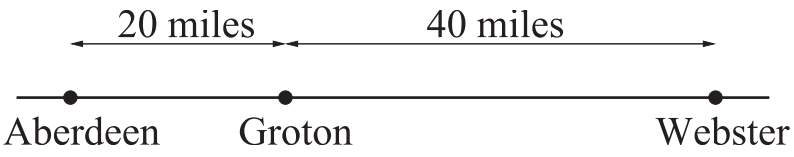


Graph 3

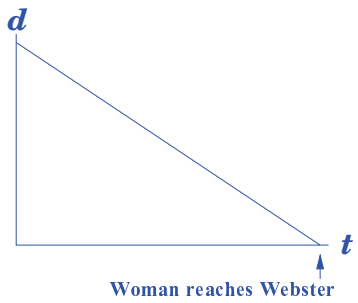


1. In Graph 1, y is a function of x because every input value of x has exactly one corresponding output value, y.
2. In Graph 2, y is not a function of x, because if it were, we would have $f(0) = 2$ and $f(0) = -2$, which is impossible. Graphically, this occurs because Graph 2 fails the vertical line test.
3. In Graph 3, y is a function of x because every input value of x has exactly one corresponding output value, y.

Example. A woman drives from Aberdeen to Webster, South Dakota, going through Groton on the way, traveling at a constant speed for the whole trip. (See map below).

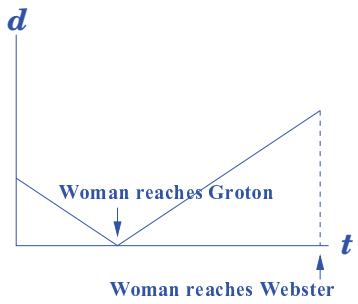


a. Sketch a graph of the woman's distance from Webster as a function of time.



Comment: Note that the function touches the t-axis; that is, $d = 0$, when the woman reaches Webster, not at $t = 0$. The constant slope of the graph indicates that her speed is constant for the duration of the trip.

b. Sketch a graph of the woman's distance from Groton as a function of time.



Comment: Note that the function crosses the t-axis; that is, $d = 0$, when the woman reaches Groton. Also, d is positive both when the woman is in Aberdeen and when the woman is in Webster, since both cities lie a positive distance away from Groton.