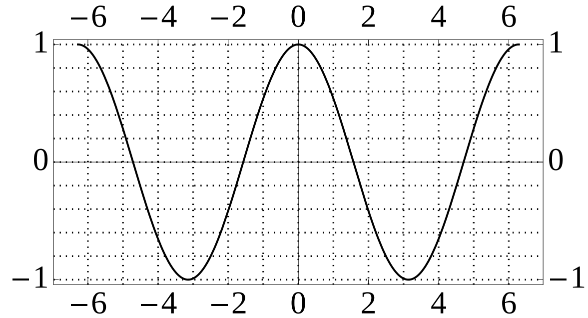
Section 9.1 – Trigonometric Equations

Example 1. Consider the graph of $y = \cos\theta$ given to the right.



(a) Graphically estimate the solutions to $\cos\theta = -0.2$, where $-2\pi \leq \theta \leq 2\pi$. (**Note:** Angles, θ, are on the horizontal axis)

(b) Use your calculator to find $\cos^{-1}(-0.2)$, and compare with your answers to (a).

(c) Use your calculator to find $\cos^{-1}(0.2)$, and use this number and symmetry to obtain more accurate solutions to $\cos\theta = -0.2$.

Example 2. For each equation, find all solutions between 0 and 2π, giving exact solutions where possible.

(a) $\sin x = -\frac{7}{10}$ (b) $6\cos x - 1 = 2$

Example 3. Find all solutions between 0 and 2π to $\tan x = -\sqrt{3}$ exactly.

Example 4. The height of a rider on a ferris wheel is given by $h(t) = 12 - 10\cos(2\pi t)$ meters, where t gives time, in minutes, after the person boards the ferris wheel.

(a) Find the amplitude, midline, and period of the function h, and then sketch a graph h corresponding to the first two minutes of the ride.

(b) During the first two minutes of the ride, find the times when the rider has a height of 10 meters.

Examples and Exercises

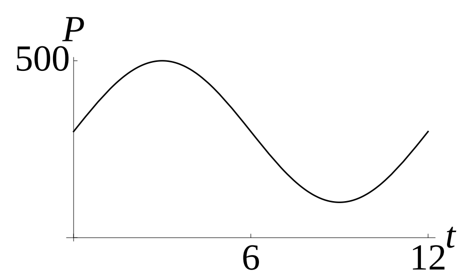
1. Find all solutions between 0 and 2π to each of the following equations, giving exact solutions where possible.

   (a) $\sin x = \frac{1}{2}$ (b) $2\tan x - 5 = 1$ (c) $\cos x = -\frac{\sqrt{2}}{2}$

2. An animal population is modeled by the function

$$P = 300 + 200\sin\left(\frac{\pi}{6}t\right)$$

   (see graph to the right), where t represents the time, in days, after the beginning of January 1st. Find the times at which the animal population is 150.



Section 9.2 – Identities, Expressions, and Equations

Preliminary Example. Discuss the difference between the equation (a) $\sin\theta = \cos\theta$ and the equation (b) $\sin(2\theta) = 2\sin\theta\cos\theta$

More Trigonometric Identities (continued from Section 7.7)

(8)	$\cos^2\theta$	$=$	$1-\sin^2\theta$	(12)	$\sin(2\theta)$	$=$	$2\sin\theta\cos\theta$
(9)	$\sin^2\theta$	$=$	$1-\cos^2\theta$	(13)	$\cos(2\theta)$	$=$	$\cos^2\theta-\sin^2\theta$
(10)	$\sec^2\theta$	$=$	$\tan^2\theta+1$	(14)	$\cos(2\theta)$	$=$	$1-2\sin^2\theta$
(11)	$\tan(-\theta)$	$=$	$-\tan\theta$	(15)	$\cos(2\theta)$	$=$	$2\cos^2\theta-1$

Example 1.

(a) Use appropriate identities from Section 8.3 to prove Identity (11) above.

(b) Divide both sides of Identity (9) by $\cos^2\theta$ and simplify to obtain Identity (10).

Example 2. Are any of the following identities? If so, prove them algebraically.

(a) $\cos\left(\dfrac{1}{x}\right) = \dfrac{1}{\cos x}$

(b) $2\tan x\cos^2 x = \sin(2x)$

Example 3. Rewrite the expression $\sec\theta - \cos\theta$ so that your final answer is a product of two trig functions.

Example 4. Solve the equation $2\sin^2\theta = 3\cos\theta + 3$ for $0 \leq \theta \leq 2\pi$.

Examples and Exercises

1. (a) Simplify $\sin A(\csc A - \sin A)$ so that your final answer is a single trig function raised to a power.

   (b) Simplify $\dfrac{1-\cos^2\theta}{\cos\theta}$ so that your answer is a product of two trig functions with no fractions.

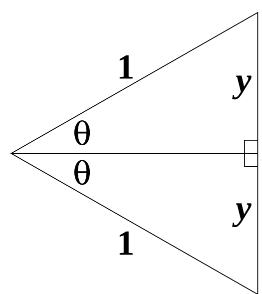
   (c) Simplify $\dfrac{\cos(2t)}{\cos t + \sin t}$ so that your answer is the difference between two trig functions with no fractions. (**Hint:** Use Formula 13 from Section 8.3 and then factor the numerator.)

   (d) Simplify $\dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta}$ so that your answer is a constant multiple of one trig function, with no fractions.

2. Find all solutions to $5\sin^2 x + \sin x = \cos^2 x$ that lie in the interval $0 \le x < 2\pi$.

3. Consider the diagram to the right.

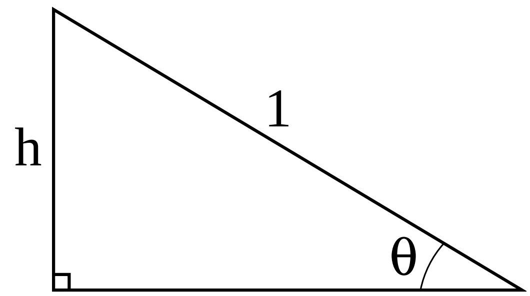
   (a) Use right triangle trigonometry to find an expression for y in terms of θ.



   (b) Prove the identity $\cos(2\theta) = 1 - 2\sin^2\theta$. Do this by applying the Law of Cosines to the "outer" triangle in the diagram; that is, the triangle with sides of length 1, 1, and $2y$.

4. Given the right triangle to the right, write each of the following quantities in terms of h.

   Note. Asking you to "write something in terms of h" is NOT asking you to solve for h. It is asking you to rewrite the quantity that you are given so that h is the only unknown in your answer.



(a) $\sin\theta$	(b) $\cos\theta$
(c) $\cos(\frac{\pi}{2} - \theta)$	(d) $\sin(2\theta)$
(e) $\cos(\sin^{-1} h)$	