Section 5.1 – Logarithms and Their Properties

Definition. If x is a positive number, then

1. $\log x$ is
2. $\ln x$ is

In other words,

$y = \log x$ means that

$y = \ln x$ means that

Example 1. Calculate the following, exactly if possible.

(a) $\log 100$

(b) $\log 0.1$

(c) $\ln \mathrm{e}$

(d) $\ln(\mathrm{e}^2)$

(e) $\log 5$

Facts about Logarithms.

1. (a) $\log(10^x) =$ $\qquad$ $\ln(\mathrm{e}^x) =$

   (b) $10^{\log x} =$ $\qquad$ $\mathrm{e}^{\ln x} =$

2. (a) $\log(ab) =$ $\qquad$ $\ln(ab) =$

   (b) $\log(\frac{a}{b}) =$ $\qquad$ $\ln(\frac{a}{b}) =$

   (c) $\log(b^t) =$ $\qquad$ $\ln(b^t) =$

3. (a) $\log 1 =$ $\qquad$ $\ln 1 =$

   (b) $\log 10 =$ $\qquad$ $\ln \mathrm{e} =$

Example 2. Solve each of the following equations for x.

(a) $5 \cdot 4^x = 25$

(b) $5x^4 = 25$

Examples and Exercises

1. Solve each of the following equations for x.

   (a) $5 \cdot 3^x = 2 \cdot 7^x$

   (b) $10\mathrm{e}^{4x+1} = 20$

   (c) $a \cdot b^t = c \cdot d^{2t}$

   (d) $5x^9 = 10$

   (e) $\mathrm{e}^{2x} + \mathrm{e}^{2x} = 1$

   (f) $\ln(x+5) = 10$

2. Simplify each of the following expressions.

(a) $\log(2A) + \log(B) - \log(AB)$

(b) $\ln(ab^t) - \ln((ab)^t) - \ln a$

3. Decide whether each of the following statements are true or false.

(a) $\ln(x+y) = \ln x + \ln y$

(b) $\ln(x+y) = (\ln x)(\ln y)$

(c) $\ln(ab^2) = \ln a + 2\ln b$

(d) $\ln(ab^x) = \ln a + x\ln b$

(e) $\ln(1/a) = -\ln a$

Section 5.2 – Logarithms and Exponential Models

Review: Two ways of writing exponential functions:

(1) $Q = ab^t$ (2) $Q = ae^{kt}$

Example 1. Fill in the gaps in the chart below, assuming that t is measured in years:

Formula		**Growth or Decay Rate**	
$Q = ab^t$	$Q = ae^{kt}$	Per Year	Continuous Per Year
	$Q = 6e^{-0.04t}$		
$Q = 5(1.2)^t$			
$Q = 10(0.91)^t$			

Example 2. The population of a bacteria colony starts at 100 and grows by 30% per hour.

(a) Find a formula for the number of bacteria, P, after t hours.

(b) What is the doubling time for this population; that is, how long does it take the population to double in size?

(c) What is the continuous growth rate of the colony?

Definition. The half-life of a radioactive substance is the amount of time that it takes for

Example 3. The half-life of a Twinkie is 14 days.

(a) Find a formula for the amount of Twinkie left after t days.

(b) Find the daily decay rate of the Twinkie.

Examples and Exercises

1. Scientists observing owl and hawk populations collect the following data. Their initial count for the owl population is 245 owls, and the population grows by 3% per year. They initially observe 63 hawks, and this population doubles every 10 years.

   (a) Find formulas for the size of the population of owls and hawks as functions of time.

   (b) When will the populations be equal?

2. Find the half-lives of each of the following substances.

   (a) Tritium, which decays at an annual rate of 5.471% per year.

   (b) Vikinium, which decays at a continuous rate of 10% per week.

3. If 17% of a radioactive substance decays in 5 hours, how long will it take until only 10% of a given sample of the substance remains?

Section 5.3 – The Logarithmic Function

1. Consider the functions $f(x) = \ln x$ and $g(x) = \log x$.

   (a) Complete the table below.

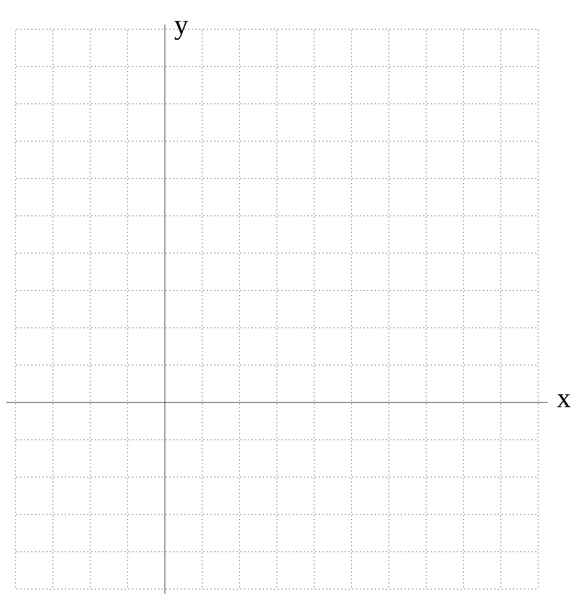
x	0.1	0.5	1	2	4	6	8	10
$\ln x$								
$\log x$								

   (b) Plug a few very small numbers x into $\ln x$ and $\log x$ (like 0.01, 0.001, etc.) What happens to the output values of each function?

   (c) If you plug in $x = 0$ or negative numbers for x, are $\ln x$ and $\log x$ defined? Explain.

   (d) What is the domain of $f(x) = \ln x$? What is the domain of $g(x) = \log x$?

   (e) Sketch a graph of $f(x) = \ln x$ below, choosing a reasonable scale on the x and y axes. Does $f(x)$ have any vertical asymptotes? Any horizontal asymptotes?



2. What is the domain of the following four functions?

    (a) $y = \ln(x^2)$
    (b) $y = (\ln x)^2$
    (c) $y = \ln(\ln x)$
    (d) $y = \ln(x - 3)$

3. Consider the exponential functions $f(x) = e^x$ and $g(x) = e^{-x}$. What are the domains of these two functions? Do they have any horizontal asymptotes? any vertical asymptotes?