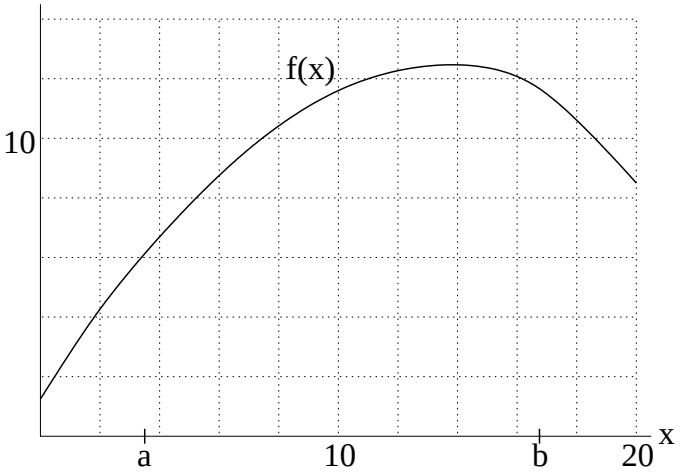
Section 2.1 – Input and Output

Preliminary Example. Complete each of the following.



1. $f(10) =$ _____.
2. If $f(x) = 10$, then $x =$ _____.
3. $f(a) =$ _____.
4. $f(10) - f(6) =$ _____.

Examples and Exercises

1. The following table shows the amount of garbage produced in the U.S. as reported by the EPA.

t (years: $1960 \equiv 60$)	60	65	70	75	80	85	90
G (millions of tons of garbage)	90	105	120	130	150	165	180

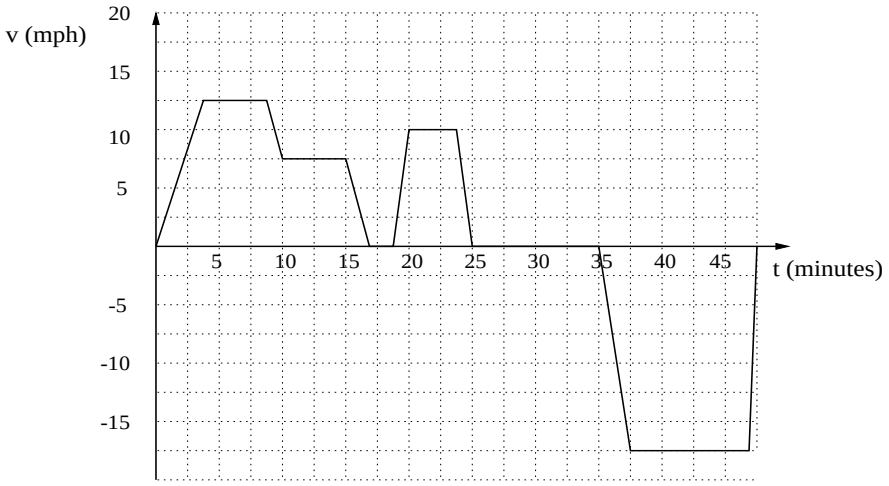
Consider the amount of garbage G as a function of time $G = f(t)$. Include units with your answers.

(a) $f(60) =$

(b) $f(75) =$

(c) Solve $f(t) = 165$.

2. Given is the graph of the function $v(t)$. It represents the velocity of a man riding his bike to the library and going back home after a little while. A negative velocity indicates that he is riding toward his house, away from the library.



Evaluate and interpret:

(a) $v(5) =$

(b) $v(40) =$

(c) $v(12) - v(7) =$

Solve for t and interpret:

(d) $v(t) = 5$

(e) $v(t) = -10$

(f) $v(t) = v(10)$

3. Consider the functions given below.

(a) Let $f(x) = x^2 - 2x - 8$.

i. Find $f(0)$.

ii. Solve $f(x) = 0$.

(b) Let $f(x) = \dfrac{1}{x+2} - 1$

i. Find $f(0)$.

ii. Solve $f(x) = 0$.

4. Let $f(x) = \dfrac{x}{x+1}$. Calculate and simplify $f\left(\dfrac{1}{t+1}\right)$, writing your final answer as a single fraction.

Section 2.2 – Domain and Range

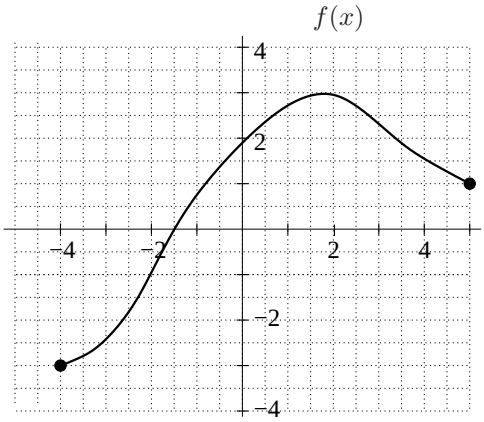
Definition. If $Q = f(t)$, then

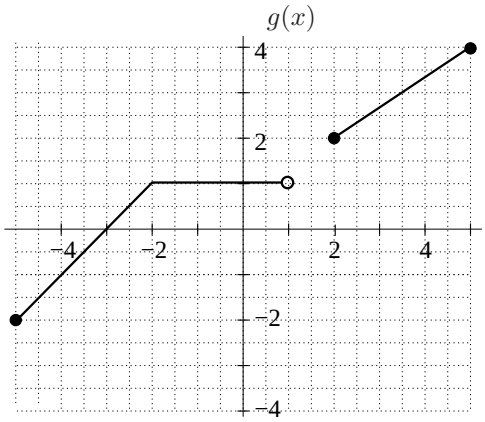
1. The _____ of f is the set of all input values, t, that yield a meaningful output value.
2. The _____ of f is the corresponding set of all output values.

Example 1. Let $A = f(r)$ be the area, in cm^2, of a circle of radius r cm. Find the domain and the range of f.

Example 2. Find the domain and range of the function $f(x) = \sqrt{x+2}$.

1. For each of the following functions below, give the domain and the range.





2. Oakland Coliseum is capable of seating 63,026 fans. For each game, the amount of money that the Raider's organization makes is a function of the number of people, n, in attendance. If each ticket costs \$30.00, find the domain and range of this function. Sketch its graph.

3. Find the domain and range of each of the following functions.

(a) $f(x) = \sqrt{3x + 7}$

(b) $g(x) = \dfrac{1}{(x-1)^2}$

(c) $h(x) = x^2 - x - 6$

(d) $k(x) = \sqrt{x^2 - x - 6}$

Section 2.5 – Preview of Inverse Functions

Preliminary Example. Recall the phone example from earlier, where a calling plan charged us a \$30 monthly service fee and then \$0.10 per minute for long distance calls.

t	0	30	33	36	60
C	30	33	33.30	33.60	36

For each of the following, fill in the blank and then give an interpretation of the entire statement.

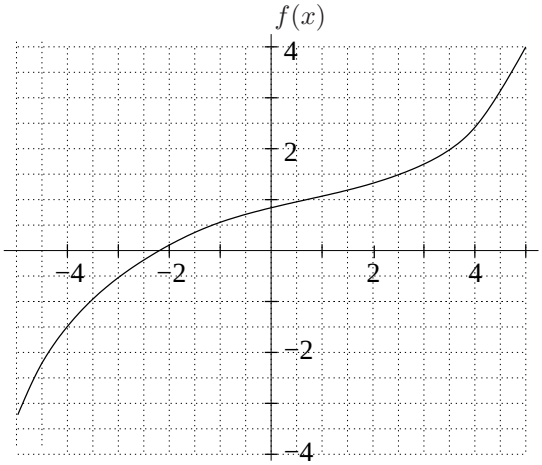
(a) $f(36) =$ _____

(b) $f^{-1}(36) =$ _____

(c) $f^{-1}($_____$) = 33$

Examples and Exercises

1. Use the two functions shown below to fill in the blanks to the right.



x	-6	-4	-2	0	2	4	6
$g(x)$	2	0	3	7	6	1	5

(a) $f(2) =$ _____ (b) $f^{-1}(2) =$ _____

(c) $g(0) =$ _____ (d) $g^{-1}(0) =$ _____

(e) $f(3) + 1 =$ _____ (f) $f^{-1}(3) + 1 =$ _____

(g) $f(3 + 1) =$ _____ (h) $f^{-1}(3 + 1) =$ _____

(i) If $g^{-1}(x) = 0$, then $x =$ _____.

2. Let $A = f(n)$ be the amount of periwinkle blue paint, in gallons, needed to paint n square feet of a house. Explain in practical terms what each of the following quantities represents. Use a complete sentence in each case.

   (a) $f(20)$

   (b) $f^{-1}(20)$

3. If a cricket chirps R times per minute, then the outside temperature is given by $T = f(R) = \frac{1}{4}R + 40$ degrees Fahrenheit.

   (a) Find a formula for the inverse function $R = f^{-1}(T)$.

   (b) Calculate and interpret $f(50)$ and $f^{-1}(50)$.

Section 2.5 – Concavity

Definitions.

1. A function $f(x)$ is called *increasing* if its graph _____ from left to right. It is called *decreasing* if its graph _____ from left to right.
2. A function $f(x)$ is called *concave up* if its average rate of change increases from left to right.
3. A function $f(x)$ is called *concave down* if its average rate of change decreases from left to right.

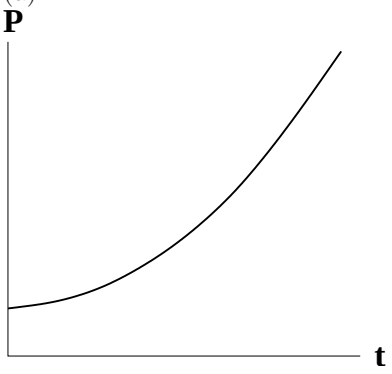
Describe the shape of the graph of a function $f(x)$ that is concave up:	Describe the shape of the graph of a function $f(x)$ that is concave down:

Example. Read the following description of a function. Then, decide whether the function is increasing or decreasing. What does the scenario tell you about the concavity of the graph modeling it?

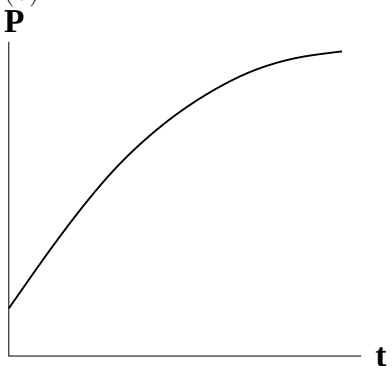
> "When a new product is introduced, the number of people who use the product increases slowly at first, and then the rate of increase is faster (as more and more people learn about the product). Eventually, the rate of increase slows down again (when most people who are interested in the product are already using it)."

Example. Consider the following graphs of population, P, as a function of time, t.

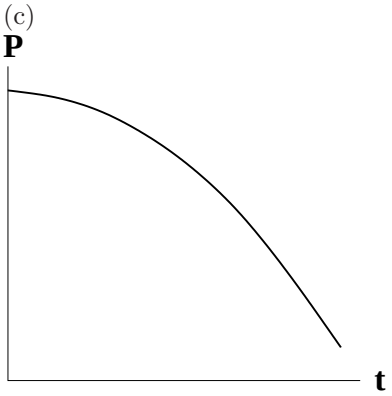
(a)



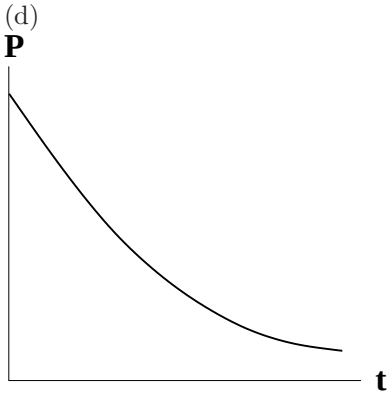
(b)



(c)



(d)



Descriptions

(a) P is _____, and the rate of change of P is _____.

(b) P is _____, and the rate of change of P is _____.

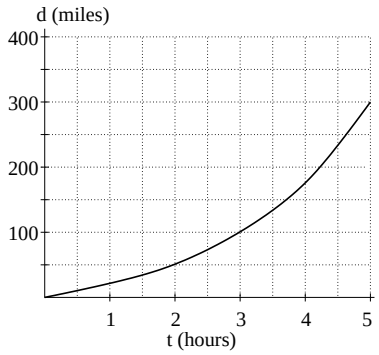
(c) P is _____, and the rate of change of P is _____.

(d) P is _____, and the rate of change of P is _____.

Examples and Exercises

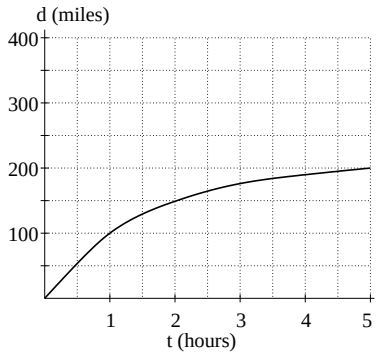
1. Consider the functions shown below. Fill in the accompanying tables and then decide whether each function is increasing or decreasing, and whether it is concave up or concave down.

(a) **Description.** This graph gives distance driven as a function of time for a California driver.



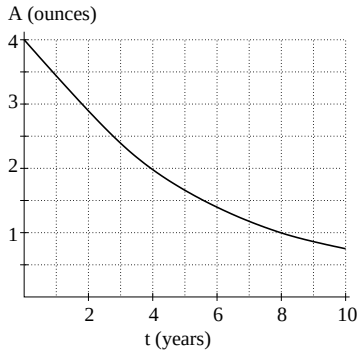
t	0	2	3	5
d				
$\frac{\Delta d}{\Delta t}$				

(b) **Description.** This graph gives distance driven as a function of time for a South Dakota driver.



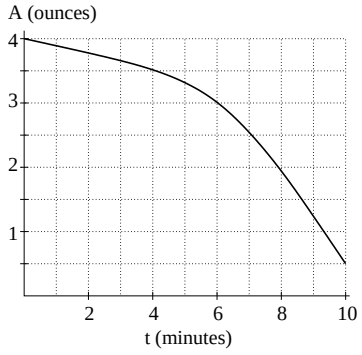
t	0	2	3	5
d				
$\frac{\Delta d}{\Delta t}$				

(c) **Description.** This graph gives the amount of a decaying twinkie as a function of time.



t	0	4	6	10
A				
$\frac{\Delta A}{\Delta t}$				

(d) **Description.** This graph gives the amount of ice remaining in a melting ice cube as a function of time.



t	0	4	6	10
A				
$\frac{\Delta A}{\Delta t}$				

2. Decide whether each of the following functions are concave up, concave down, or neither.

x	0	1	2	3	4
$f(x)$	1	3	6	10	20

x	0	1	2	3	4
$g(x)$	10	9	7	4	0

$p(x) = 3x + 1$