Chapter 1 Skills – Points and Linear Equations

Example 1. Solve $\quad \dfrac{2}{2-t} = \dfrac{3}{2-2t} \quad$ for t.

Example 2. Solve $\quad 2 = \dfrac{A+Bt}{A-Bt} \quad$ for t.

Example 3. Write $\dfrac{A}{C+Bt}+\dfrac{B}{D-Ct}$ as a single fraction.

Example 4. Solve $\begin{array}{rcl} 4x+3y & = & 11 \\ 2x-\frac{y}{3} & = & 0 \end{array}$ for x and y.

Section 1.1 – Functions and Function Notation

Definition. A *function* is a rule that takes certain values as inputs and assigns to each input value exactly one output value.

Example. Let $y = \dfrac{2}{1+x}$.

x	0	1	2	3
y				

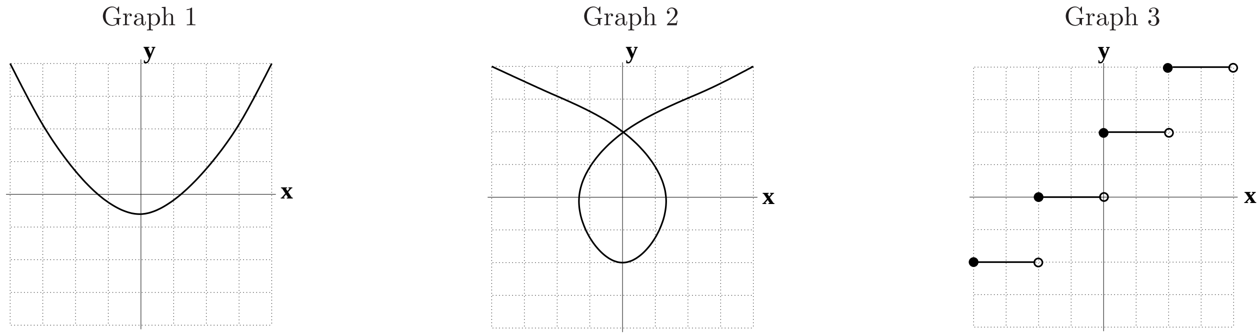
Example.

$$\begin{aligned} t &= \text{time (in years) after the year 2000} \\ w &= \text{number of San Francisco '49er victories} \end{aligned}$$

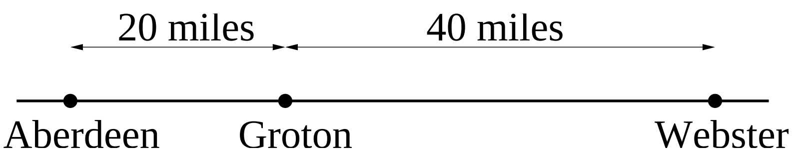
t	0	1	2	3	4	5	6	7	8	9
w	6	12	10	7	2	4	7	5	7	8

Observations:

Example. Which of the graphs below represent y as a function of x?



Example. A woman drives from Aberdeen to Webster, South Dakota, going through Groton on the way, traveling at a constant speed for the whole trip. (See map below).



a. Sketch a graph of the woman's distance from Webster as a function of time.

b. Sketch a graph of the woman's distance from Groton as a function of time.

Section 1.2 – Rates of Change

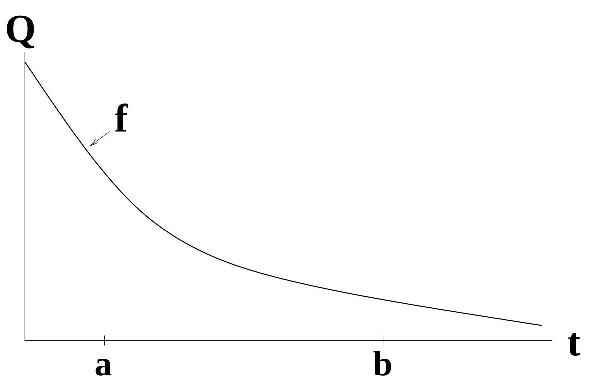
Preliminary Example. The table to the right shows the temperature, T, in Tucson, Arizona t hours after midnight.

t (hours after midnight)	0	3	4
T (temp. in °F)	85	76	70

Question. When does the temperature decrease the fastest: between midnight and 3 a.m. or between 3 a.m. and 4 a.m.?

Graphical Interpretation of Rate of Change

Definition. The **average rate of change**, or just **rate of change** of Q with respect to t is given by

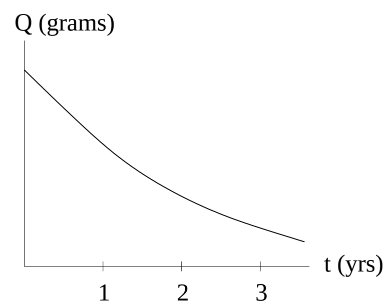


Alternate Formula for Rate of Change: The average rate of change of a function $Q = f(t)$ on the interval $a \leq t \leq b$ is given by the following formula:

Examples and Exercises

1. Let $f(x) = 4 - x^2$. Find the average rate of change of $f(x)$ on each of the following intervals.
   (a) $0 \leq x \leq 2$ (b) $2 \leq x \leq 4$ (c) $b \leq x \leq 2b$

2. To the right, you are given a graph of the amount, Q, of a radioactive substance remaining after t years. Only the t-axis has been labeled. Use the graph to give a **practical interpretation** of each of the three quantities that follow. A practical interpretation is an explanation of meaning using everyday language.

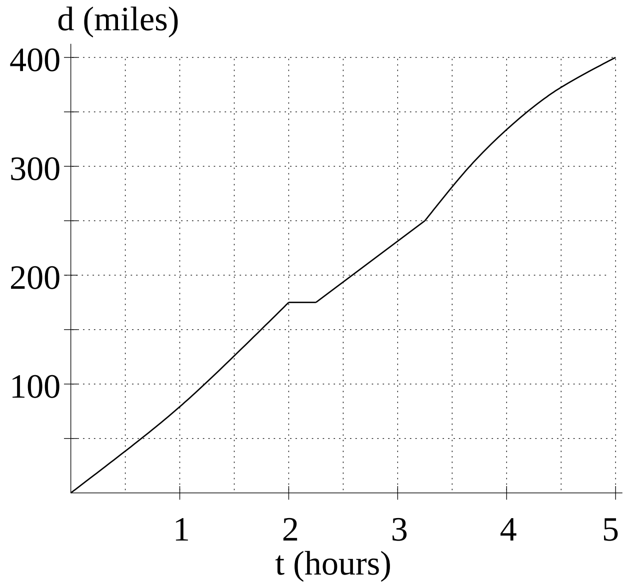


   a. $f(1)$

   b. $f(3)$

   c. $\dfrac{f(3) - f(1)}{3 - 1}$

3. Two cars travel for 5 hours along Interstate 5. A South Dakotan in a 1983 Chevy Caprice travels 300 miles, always at a constant speed. A Californian in a 2009 Porsche travels 400 miles, but at varying speeds (see graph to the right).



(a) On the axes above, sketch a graph of the distance traveled by the South Dakotan as a function of time.

(b) Compute the average velocity of each car over the 5-hour trip.

(c) Does the Californian drive faster than the South Dakotan over the entire 5 hour interval? Justify your answer!

Sections 1.3 & 1.4 – Linear Functions and Their Formulas

Preliminary Example. The cost, C, of your monthly phone bill consists of a \$30 basic charge, plus \$0.10 for each minute of long distance calls.

(a) Complete the table below, and sketch a graph.

t	0	30	60	90	120	150
C						

(b) Compute the average rate of change of C over any time interval.

(c) Find a formula for C in terms of t.

(d) If your bill is \$135, how long did you talk long distance?

Notes on Linear Functions:

1. If $y = f(x)$ is a linear function, then $y = mx + b$, where

2. If $y = f(x)$ is linear, then ______________________ input values produce output values.

Different forms for equations of lines:

Example 2. Find the slope and the y-intercept for each of the following linear functions.

(a) $3x + 5y = 20$

(b) $\dfrac{x - y}{5} = 2$

Examples and Exercises

1. Let $C = 20 - 0.35t$, where C is the cost of a case of apples (in dollars) t days after they were picked.

   (a) Complete the table below:

t (days)	0	5	10	15
C (dollars)				

   (b) What was the initial cost of the case of apples?

   (c) Find the average rate of change of C with respect to t. Explain in practical terms (i.e., in terms of cost and apples) what this average rate of change means.

2. In parts (a) and (b) below, two different linear functions are described. Find a formula for each linear function, and write it in slope intercept form.

   (a) The line passing through the points $(1, 2)$ and $(-1, 5)$.

   (b)

C	10	15	20	25
F	50	59	68	77

3. According to one economic model, the demand for gasoline is a linear function of price. If the price of gasoline is $p = \$3.10$ per gallon, the quantity demanded in a fixed period of time is $q = 65$ gallons. If the price is \$3.50 per gallon, the quantity of gasoline demanded is 45 gallons for that period.

   (a) Find a formula for q (demand) in terms of p (price).

   (b) Explain the economic significance of the slope in the above formula. In other words, give a practical interpretation of the slope.

   (c) According to this model, at what price is the gas so expensive that there is no demand?

   (d) Explain the economic significance of the vertical intercept of your formula from part (a).

4. Look back at your answer to problem 2(b). You might recognize this answer as the formula for converting Celsius temperatures to Fahrenheit temperatures. Use your formula to answer the following questions.

   (a) Find C as a function of F.

   (b) What Celsius temperature corresponds to 90°F?

   (c) Is there a number at which the two temperature scales agree?

Sections 1.4 & 1.5 – Geometric Properties and Modeling with Linear Functions

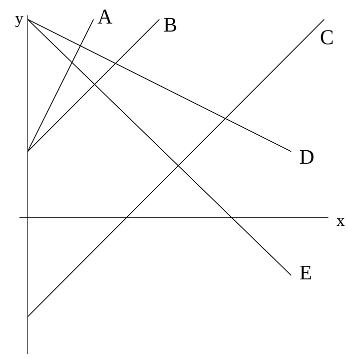
Example 1. You need to rent a car for one day and to compare the charges of 3 different companies. Company I charges \$20 per day with an additional charge of \$0.20 per mile. Company II charges \$30 per day with an additional charge of \$0.10 per mile. Company III charges \$70 per day with no additional mileage charge.

(a) For each company, find a formula for the cost, C, of driving a car m miles in one day. Then, graph the cost functions for each company for $0 \leq m \leq 500$. (Before you graph, try to choose a range of C values would be appropriate.)

(b) How many miles would you have to drive in order for Company II to be cheaper than Company I?

Example 2. Given below are the equations for five different lines. Match each formula with its graph to the right.

- $f(x) = 20 + 2x$
- $g(x) = 20 + 4x$
- $h(x) = 2x - 30$
- $u(x) = 60 - x$
- $v(x) = 60 - 2x$



Facts about the Line $y = mx + b$

1. The y-intercept, b (also called the vertical intercept), tells us where the line crosses the ______________.
2. If $m > 0$, the line ______________ left to right. If $m < 0$, the line ______________ left to right.
3. The larger the value of $|m|$ is, the ______________ the graph.

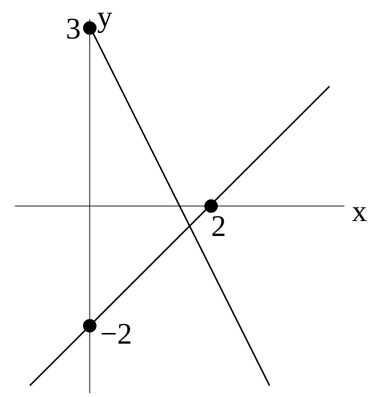
Parallel and Perpendicular Lines

<u>Fact</u>: Two lines ($y = m_1x + b_1$ and $y = m_2x + b_2$) are ...

1. ... <u>parallel</u> if ______________
2. ... <u>perpendicular</u> if ______________

Examples and Exercises

1. Consider the lines given in the figure to the right. Given that the slope of one of the lines is -2, find the **exact coordinates** of the point of intersection of the two lines. ("Exact" means to leave your answers in fractional form.)



2. Parts (a) and (b) below each describe a linear function. Find a formula for the linear function described in each case.

   (a) The line parallel to $2x-3y=2$ that goes through the point $(1,1)$.

   (b) The line perpendicular to $2x-3y=2$ that goes through the point $(1,1)$.